

Assessing Lower Secondary School Students' Common Errors in Statistics

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ABSTRACT

Statistical literacy has been emphasised in the school mathematics curriculum, with the growing concern about students' ability to think critically in solving statistical problem-solving tasks. However, the current studies revealed that secondary school students' errors mainly involve the problem of basic concepts in statistics, data interpretation, and the selection of an appropriate representation of data. Therefore, this study aimed to analyse the common errors made by students in solving statistics tasks with multi-level complexity. A survey method was applied in this study. The sample of this study consisted of 356 Form One (Grade 7) students from eight secondary schools. The instrument of this study consisted of five superitem tasks, which represented the five content domains: line graph, bar graph, pie chart, dot plot, and histogram. There are four levels of items in each superitem task. Thus, the total number of items is 20. The format of all the 20 items in the five superitem tasks is open-ended. The common errors were then analysed based on all the participants' solutions shown in their answer script. The findings found that most students could not achieve the highest level of statistical competency. They failed to think qualitatively while justifying data. This study provides a meaningful analysis that assists the teaching and learning of statistics to better link numeracy and literacy. The application of the superitem tasks provides valuable information that enables the teachers to understand their students' statistical processes better.

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INTRODUCTION

Statistics involves collecting, interpreting, analysing, and making inferences about the data (Idehen, 2020). Many daily life activities require understanding statistical

data to make decisions related to health, such as the COVID-19 pandemic statistical data, finance, employment, sport, and advertisement. Hence, the need to develop statistical literacy has been emphasised in the school mathematics curriculum, with the growing concern about the student's ability to think critically and creatively in solving statistical problem-solving tasks (Thongoon et al., 2021). The National Council of Teachers of Mathematics (NCTM, 2020) draws attention to the increasing importance of middle school students' statistical literacy. NCTM (2020) suggested that the students are expected to formulate research questions, design a study, collect the data, use appropriate graphical representation, understand and discuss the data sets, and develop inferences and predictions based on the data.

Similarly, the Malaysian lower secondary school, namely Form One (grade seventh) students (13 years old), are also expected to learn data representation and interpretation in the context of complex routine problem-solving. Meanwhile, in Form Two, they learn and apply the concept of central tendency in the context of non-routine problem-solving (Kementerian Pendidikan Malaysia, 2015, 2016). The non-routine problem solving requires some creativity and does not have a definite answer or solution. It can be solved with multiple strategies.

Even though the ability to solve statistical problem-solving tasks is important, literature findings show that the solution of statistical tasks has not been as

expected. It was raised by Idehen (2020) and Saidi and Siew (2019) that errors faced mainly by secondary school students involve the problem of basic concepts in statistics, data interpretation, and the selection of appropriate representation. Chan et al. (2016), Foo (2017), as well as Saidi and Siew (2019) found that the learning of statistical concepts among Malaysian students is unlikely to be achieved. Most of them harbour misconceptions and difficulties in learning various topics in statistics (Ibnatul et al., 2021). Saidi and Siew (2019) revealed that most Malaysian secondary school students have a low understanding of measures of central tendency properties and a very low understanding of the problem and data representation. The finding indicates that the students did not understand the mode concept and were confused with mean, mode, and median. As a result, they made various errors and faced problems in (i) applying the measures of central tendency when the data was in quantitative or qualitative form and (ii) choosing which type of measure of central tendency was the best representative for the given data.

Similar errors have also been documented in elementary and undergraduate school students (Ibnatul et al., 2021; Lynch et al., 2000). Angateeah (2017) and Reaburn (2011) claimed that students' errors in solving mathematics tasks are caused by many reasons, namely misunderstanding of concepts, carelessness of calculation, and wrong application of operation or formula. As a result, mathematical error problems have become a great concern to teachers,

students, parents, and policymakers. However, most of the previous studies focused more on the student's achievement and failure in certain statistical topics alone without informing the reasons for failure in answering the tasks. Even though there are a few studies (Fitriyah et al., 2020; Idehen, 2020; Sari & Bernard, 2020) that focused on the investigation of students' common errors in statistics, the errors were analysed based on the solutions shown by students in solving a particular problem-solving task, either through paper-and-pencil tests or interviews.

Some limitations have been identified in these existing studies. First, the application of the interview method only involved a small sample, and it was very time-consuming to analyse the errors. Second, none of the previous studies provided detailed information about the errors made by the students while solving multi-level complexity tasks. A mathematical problem-solving task normally requires multiple solutions steps to achieve the answer. It challenges the students' conceptual understanding and procedural skills. Thus, detailed information is needed to inform the teachers and students about the various errors in solving the task from the basic level to the highest level. The students who can easily detect their weaknesses at the basic level will increase the possibility of responding correctly at a higher task level. Based on these limitations, this study aimed to analyse the common errors made by students in solving statistical tasks with a multi-level of complexity.

Objectives

The objectives of the study were as follows:

1. To develop the statistical tasks with a multi-level of complexity based on the cognitive development model, namely the SOLO model.
2. To analyse the common errors in solving the statistical tasks based on Newman's Error Analysis.

LITERATURE REVIEW

One of the common models used to analyse mathematical errors is Newman's Error Analysis (Newman, 1983). Newman's error analysis provides five stages of analysis of the mathematical mistakes made by students. According to Newman, when students solve a standard mathematical word problem, they must pass through five stages of consecutive hurdles, namely reading, comprehension, transformation, process skill, and encoding. Newman believed that failure at any stage would prevent the students from getting accurate solutions.

Students often cannot read the mathematical task correctly or the important information incorrectly at the reading stage. At the comprehension stage, students show that they do not understand the task or may not understand the specific terms in the task. Students cannot transfer the task to the appropriate mathematical strategy or model at the transformation stage. They fail to select the appropriate mathematical operation or model to represent the data, such as a graph or chart. Although the correct mathematical strategy and data representation have been appropriately selected at the process skill

stage, the calculation and the solution steps are inaccurate or missing. Students often cannot write acceptable and complete responses at the encoding stage. As a result, they cannot write and express ideas logically and critically. This study applied this model to identify the students' common errors in solving statistical tasks with multi-level items.

Newman's Error Analysis was used to identify and analyse the students' mathematical word problems. For instance, Chin and Lim (2018) and Fitriani et al. (2018) described the students' errors in solving algebraic tasks. Furthermore, Fitriani et al. (2018) analysed the students' errors while solving the derivative of function problem. Data were collected through problem-solving tests and interviews of senior high school students (Grade 11) in Bandung. The results showed that students made five types of errors in solving the problem of derivatives of algebraic functions which were comprehension error, transformation error, process skill error, an encoding error, and careless error.

Haryanti et al. (2019) identified the students' errors in solving the word problems with plane geometry. 23 grade 7 students from a Junior High School in Subang, Indonesia, were interviewed. The results showed that most students made mistakes in transforming the word problem related to plane geometry into a mathematical model—formulas and illustrations with pictures. The ability to calculate operations was the most common error in the student's answers. Meanwhile, Khalo and Bayaga

(2015) identified the errors committed by learners in financial mathematics and why the learners continued to make such errors. There were 105 Grade 10 mathematical literacy learners involved. The structured interview questionnaire was used for collecting the data. The content and correlation analysis revealed that learners tend to forget to read the instructions and round off incorrectly. However, to date, no research on providing detailed information about the errors made by students while solving multi-level complexity tasks has been reported in the literature.

In the process of developing the assessment framework and instrument, the information processing theory developed by Craik and Lockhart (1972) was implemented. This theory emphasises the importance of deep processing information, which leads to a greater understanding of the concept learned. It believes that deep information processing contributes to a better understanding of the concept and academic achievement. The learners need to have the ability to solve the surface-level items before progressing to the deep-level items (Smith & Colby, 2007). Hence, the assessment should include a balance of surface and deep-level items. The SOLO (Structure of the Observed Learning Outcome) model is the established and famous model ensuring the assessment covers surface and deep level items. It plays an important role as an assessment model that values the balance of surface and deep processing (Hattie & Brown, 2004; Huan & Melissa, 2018). The development

of the assessment tasks in this study was based on the SOLO model and the idea of a superitem format. The SOLO model was developed by Biggs and Collis (1982). This model emphasises the concept of cumulative cognitive dimension and latent hierarchy. The rationale for using this combination is to produce more user-friendly and practicable tasks that can easily diagnose and identify student errors at each level. The format of the superitem task consists of two components. The stem is the first component. It represents the scenario or problem in paragraph form. The second component consists of the four-level items representing the SOLO model's four main levels.

The assessment framework's content domains and statistical processes, the lower secondary school (grade seven) mathematics curriculum were referred to. The five content domains involved were line graph, bar graph, pie chart, dot plot, and histogram. In addition, four statistical processes were identified, representing the main cognitive processes when engaging the data handling: understanding the data provided, calculating and comparing the value of data, representing the data into various types, and making inferences and predictions. The middle school curriculum covers these statistical processes in most countries (Thong-oon et al., 2021; Van de Walle et al., 2014). These four statistical processes were assessed across four levels of cognitive development based on the SOLO model: uni-structural, multi-structural, relational, and extended abstract. It means that four levels of items had been developed

in each task to assess the statistical processes hierarchically. For example, the item can be easily responded to at the uni-structural level by identifying single information provided in the task's stem.

For instance, the pie chart states the number of students who go to school by car. The item can be responded to at the multi-structural level by referring to more or all the information in the stem. It may even involve some basic mathematical skills to respond. For instance, the student compares the values supplied in the pie chart to identify the highest number. The item can be responded to at the relational level by relating all the relevant aspects of data and converting or representing the data in the appropriate graphical form. For instance, the students construct the pie chart based on the information in the table. At the highest level, the students must infer and predict through analytic and logical thinking based on their existing knowledge.

In short, students' ability to respond correctly at a certain item level indicated their statistical ability. Therefore, the errors were analysed based on Newman's Error Analysis when the students were stuck at a certain level and unable to achieve the higher levels.

METHODOLOGY

A survey method was applied in this study. The sample of this study consisted of 356 Form One (Grade 7) students from eight secondary schools in Penang State, Malaysia. The sample was selected from the high, middle, and low-performance

classes to ensure that the findings represent the population's standard performance. In addition, their latest school-based mathematics test results were used to determine the student's performance levels. The instrument of this study consisted of five superitem tasks, which represented the five content domains: line graph, bar graph, pie chart, dot plot, and histogram. The content domains were based on the main learning standard of the data handling topic in the Malaysian Secondary School Form One Mathematics KSSM Standard-Based Curriculum, namely, constructing the data representation, including bar charts, pie charts, line graphs, dot plots, stem and leaf plot and histogram (Kementerian Pendidikan Malaysia, 2015).

There were four level of items in each superitem task. Thus, the total number of items is 20. The format of all the 20 items in the five superitem tasks is open-ended. The development of the superitem tasks involved three main phases:

(i) develop the assessment framework. The Malaysian Secondary School Form One Mathematics KSSM Standard-Based Curriculum (Kementerian Pendidikan Malaysia, 2015) and the features of SOLO levels were the main sources for identifying the statistical processes across the topic's content. Four statistical processes had been determined: understanding the data provided, calculating and comparing the value of data, representing the data into various types, and making inferences and predictions;

(ii) developing five superitem tasks based on the assessment framework. Based on the example of the superitem task (Appendix 1), only a value in the diagram needs to be referred to respond at the first level of the item. For instance, identify the number of students enrolled in the year 2016 to give the correct response. At the second level of the item, two or more values in the diagram need to be referred to identify the range. For instance, identify the number of students enrolled in 2015 and 2019, then calculate the difference between them. At the third level of the item, all the values shown in the diagram need to be analysed and converted into a new graphical form. Finally, prediction and logical reasoning are required based on the new data representation at the last level of the item. Table 1 shows the content domain of five superitem tasks based on the SOLO model.

(iii) the content-based validity evidence had been determined by five experts in their area of specialisation, namely the experienced Form One mathematics teachers and mathematics education lecturers. The Item-CVI (I-CVI) and Scale-level CVI (S-CVI) were determined to quantify the judgment data. The result of I-CVI was between 0.8 to 1.0, indicating that all the items were within the acceptable range (Polit et al., 2007). The S-CVI was 0.93, indicating that the superitem tasks were also within the acceptable range of S-CVI. The construct-based validity

Table 1
The content domain of five superitem tasks based on the SOLO model

Superitem	Unistructural (reading the data)	Multistructural (reading between the data)	Relational (representing data)	Extended abstract (reading beyond the data)
1	Refer to a single value of the diagram to give a response	find the difference between the two values.	Represent the information into a line graph	Make a prediction and provide a logical reason based on the existing knowledge and the information in the stem
2		find the value in percentage	Represent the information in a bar chart	
3		find the highest value	Represent the information into a pie chart	
4		find the total value.	Represent the information into a dot plot	
5		find the value in percentage.	Represent the information in a histogram	

evidence was also determined based on the Principles of Rasch Model, namely the unidimensionality, item fit, item polarity, and reliability separation indices. Based on the findings, the newly developed assessment tool had fulfilled the four main components stated in the Rasch Model analysis.

Appendix 1 shows an example of a line graph superitem task developed in this study. The students were given one hour and thirty minutes to answer the five superitem tasks. The students were required to show all their solutions in the space provided. The collected data were analysed using both quantitative and qualitative approaches. The quantitative data were analysed for the instrument's descriptive analysis, reliability, and validity. The focus of this paper was to discuss the common errors in solving statistical problem-solving tasks in depth. Therefore, more emphasis was placed on the qualitative data analysis. The common errors were then analysed based on all the participants' solutions shown in their answer

script. First, the student's responses were evaluated using the scoring scheme.

All the possible responses were determined, and the scores were allocated for each item level in all the tasks based on rationality. For instance, 0 and 1 scores were allocated for the simplest level of the item, namely the uni-structural level, because the correct response only requires the identification of a value from the data provided. Therefore, no score was given for the incorrect response, and 1 score was given for a correct response. In addition, there were 0, 1, and 2 scores were allocated for the second level of the item (multi-structural), and 0, 1, 2, and 3 scores were allocated for the third and highest level of an item, which involved the data representation and development of inference and prediction.

Two mathematics experts validated the scoring procedure to ensure the appropriateness of the score assigned to each level of items. The experts were asked to rate the appropriateness of the scores on a 5-point scale (1 = Not appropriate, 2

= Less appropriate, 3 = Appropriate, 4 = Quite appropriate, 5 = Very appropriate). A simple per cent agreement approach was used to capture the consensus of the experts. The consensus between the two raters is considered as reached if the two raters come to an exact agreement by giving the same rate during the validation process. For each pair of experts, the simple per cent agreement was calculated by dividing the total number of exact agreements among each pair of experts by the total number of items rated by the experts (Stemler & Tsai, 2008). The simple per cent agreement among each pair of experts was 100 per cent (>70 %) and was accepted for this study (Graham et al., 2012).

FINDINGS

This study identified four stages of errors: comprehension, transformation, process skill, and encoding errors. Since the items were developed in the same hierarchical manner for each superitem task, the errors

were analysed and interpreted according to the levels: uni-structural, multi-structural, relational, and extended abstracts.

Table 2 shows the common error analysis for uni-structural level items. The uni-structural level items were the easiest. The students only need to read and refer to relevant information from the diagram to give their responses. Almost all the students could respond correctly to the items except for superitem 3 (35% of students answered incorrectly) and superitem 4 (10% of students answered incorrectly). For superitem 3, the error was identified at the process skill stage. For example, some students could state 55, but the unit of thousand was missing. For superitem 4, errors were detected at the comprehension stage. For example, students did not understand the stem-and-leaf plot. They counted the number of digits at the leaf for the stem ‘4’ and answered ‘6’ or chose the last digit of the leaf and gave the answer ‘9’.

Table 2
Common error analysis for uni-structural level items

No	Content of item	Types of error		Total number of students who answered incorrectly (n=256)	Percentage (%)
		Comprehension	Process skill		
Superitem 3a	How many Myvi cars were sold in 2017?		Ignored the unit of thousand. Respondents stated 55.	124	35
Superitem 4a	How many athletes weighed 46 kilograms (kg)?	Did not understand the stem-and-leaf plot. Examples: Counted the frequency of the leaf. Chose the last digit, namely 9		34	10

Table 3 shows the common error analysis for multi-structural level items. For the multi-structural level, the students need to refer to more or all information in the stem, then: (i) apply the mathematical concept and skills to calculate the total value, differences, or percentage; or (ii) compare the values given.

Errors were detected at the process skill stage for Superitem 1 (7% of students provided partially correct and incorrect answers). The students could apply the operation appropriately to find the difference in the number of students enrolled, but they erroneously read the figures from the bar graph. For Superitem 2 and Superitem 5, errors were identified at the transformation and process skill stages. In other words, 22% of students gave partially correct and incorrect answers in Superitem 2, and 42% of students answered partially correct and incorrect in Superitem 5. Figure 1 shows that the students used addition operation instead of multiplication to calculate the percentage in Superitem 5. They merely totalled up the frequency of students to make up a percentage value.

The students could not apply the appropriate mathematical strategy for calculating percentages at the transformation stage. Similarly, Figure 2 shows that the students calculated the mean by summing up the number of students who used different types of transport to school and applied an inappropriate strategy to find the percentage value in Superitem 2. Meanwhile, at the process skill stage, although the students could correctly represent the mathematical

strategy, they made mistakes in calculating the percentage or reading the figures from the pie chart or table. Figure 3 shows that the students could apply the appropriate formula to calculate the percentage for Superitem 2 but made a mistake in the solution steps. For Superitem 3 (3% of the students who provided partially correct and incorrect answers), the student failed to understand or misunderstand the term 'the highest number' of cars sold. As a result, they computed the total number of cars sold.

Superitem 4, errors could be detected at the comprehension stage (14% of the students offered partially correct and incorrect answers). The students did not understand the stem-and-leaf plot. The item requires the total number of athletes, but the students calculated and totalled up the weight of each athlete (refer to Figure 4). Some of them totalled up the weight and multiplied it by 2.

Table 4 shows the analysis of common errors for the relational level items. Students represented the data inappropriately at the transformation stage for the relational level. They failed to master the concept of various forms of data representation. For instance, for Superitem 1, they did not understand the concept of the line graph. Hence, they converted the graph into various forms, especially bar graphs (refer to Figure 5) (41% of the students answered partially correct and incorrect). For Superitem 2, the students did not understand the concept of a bar graph. Therefore, they converted it into different forms of data representation, especially line graphs, bar

Table 3
Common error analysis for multi-structural level items

No	Content of item	Types of error			Total number of students who answered partially correct (n=256)	Total number of students who answered incorrectly (n=256)	Percentage (%)
		Comprehension	Transformation	Process skill			
Superitem 1b	Find the difference in the number of students enrolled in 2015 and 2019			The operation applied was appropriate, but the students erroneously read the figure from the bar graph.	21	5	7
Superitem 2b	Find the percentage of the students who cycle to school	Unable to transform the problem to the correct mathematics strategy, namely unable to select the appropriate mathematical operation to find the percentage. For example: i. $100-10=90\%$ ii. $3/100 \times 360 = 120\%$ iii. $30/100 \times 3 = 0.9\%$ iv. $3/3 \times 10 = 10\%$ v. $30/100 \times 7 = 2.1\%$ vi. $30-10=20\%$ vii. $3/30=10\%$ viii. $7/30 \times 360 = 84\%$ ix. $3+8+10+7+2=30\%$ x. $3/100 \times 80 = 2.4\%$ xi. $3/30 \times 360 = 36\%$ xii. $3/24 \times 100 = 12.5\%$ xiii. $3/130 \times 100 = 10\%$ xiv. $3/30-3/30 = 27/30\%$ xv. $100/30 \times 2 = 6\%$	Made a mistake in calculating the percentage or mistake in reading the figure from the pie chart, for example: i. $3/30 \times 100 = 3\%$ ii. $3/30 \times 100 = 0.3\%$ iii. $3/30 \times 100 = 0.1\%$ iv. $3/30 \times 100 = 30\%$ v. $8/30 \times 100 = 10\%$ vi. $7/30 \times 100 = 23.3\%$	16	63	22	
Superitem 3b	Which type of car shows the highest number of cars sold in 2017?	Computed the total number of cars sold.			3	8	3

Table 3 (continue)

No	Content of item	Types of error			Total number of students who answered partially correct (n=256)	Total number of students who answered incorrectly (n=256)	Percentage (%)
		Comprehension	Transformation	Process skill			
Superitem 4b	What is the total number of athletes in the school?	Did not understand the stem-and-leaf plot. Examples: i. Total the weight of each athlete. ii. $46 + 46 + 48 + \dots = 494$ iii. Total the weight times 2. $494 \times 2 = 988$			2	45	14
Superitem 5b	Calculate the percentage of students who spent less than 30 hours for their individual study.		Unable to transform the problem into the correct mathematics strategy. For example: i. $31/100 \times 35 = 10.85\%$ ii. $31 \times 35/100 = 10.85\%$ iii. $3 + 8 + 12 + 7 + 4 = 34\%$ iv. $31/34 = 91\%$ v. $31/100 \times 360 = 111\%$ vi. $35/360 \times 100 = 9\%$	Made a mistake in calculating the percentage or mistake in reading the figure from the table. For example: $31/35 \times 100 = 25.6\%$ $21/35 \times 100 = 60\%$ $30/35 \times 100 = 85.7\%$ $31/35 \times 100 = 23\%$ $23/35 \times 100 = 65.71\%$ $8/35 \times 100 = 2.28\%$	63	86	42

$$4 + 7 + 12 + 8 = 31\%$$

Figure 1. Example of solution for 5b

$$7 + 2 + 3 + 8 + 10 = 30$$
~~$$39 + 14 = 53\%$$~~

$$\frac{30}{100} \times 7 = 2.1\%$$

Figure 2. Example of solution 2b

$$\frac{3}{34} \times 100 = 3\%$$

Figure 3. Example of solution 2b

What is the total number of athletes in the school?

$$46 + 46 + 48 + 48 + 49 + 49 = 285$$

$$50 + 52 + 52 + 52 + 54 + 54 + 56 + 56 + 58 + 58 = 542$$

$$60 + 62 = 122$$

$$285 + 542 + 122 = 949$$

Figure 4. Example of solution 4b

graphs, histograms, and tables (46% of the students answered partially correct and incorrect). Figure 6 shows that the students converted the pie chart into a table instead of a vertical bar chart.

For Superitem 4, the students did not understand the concept of the dot plot. They converted it into a line graph or created their forms of data representation (refer to Figure 7) (85% of the students answered partially correct and incorrect). For Superitem 5, the students did not understand the concept of a histogram (71% of the students answered partially correct and incorrect). They converted it into different forms of data representation, such as a bar graph, line graph, and table (refer to Figures 8, 9, and 10). Besides, the students also made errors at the process skill stage. Although they could represent the data appropriately, the axis was labelled incorrectly, the x-axis and y-axis were not labelled, the scale was incorrect, there was no line for the axis, and they failed to label the values on the axis. These similar errors appeared in Superitems 1, 2, 4, and

5. For Superitem 3, although the students could construct the pie chart, they did not label the values. Apart from that, the value of angles and the calculation of angles were also inaccurate.

Table 5 analyses common errors for the extended abstract level items. For this highest level of the item, most students could not give a complete answer. They

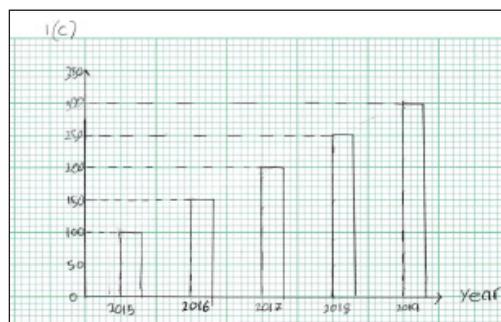


Figure 5. Example of solution for 1c

2.

Transportation	Car	School Bus	Bicy cle	Motor cycle	walk
Number of student	8	10	3	7	2

Figure 6. Example of solution 2c

Table 4
Common error analysis for relational level items

No	Content	Transformation	Process skill	Total number of students who answered partially correctly (n=256)	Total number of students who answered incorrectly (n=256)	Percentage (%)
Superitem 1c	Convert the bar chart into a line graph	Did not understand the concept of a line graph. They have converted it into various forms of graphs. For example: i. horizontal bar graph ii. vertical bar graph iii. join graph (bar + line) iv. other forms of data representation	Able to construct the line graph but: i. did not label the axis ii. the x-axis was not labelled correctly iii. the x-axis was not labelled iv. the scale was not correct v. no line for the axis vi. did not label the values on the axis	38	107	41
Superitem 2c	Convert the pie chart into a vertical bar chart.	Did not understand the concept of a bar chart. They represented the data inappropriately; converted it into various forms of graphs and charts. For example: i. line graph ii. horizontal bar graph iii. histogram iv. table	Able to construct the bar graph but: i. no label for the axis ii. the scale of the y-axis was incorrect iii. labelled the values incorrectly on the axis. iv. no axis.	114	51	46
Superitem 3c	Construct a pie chart to represent the data given.		Able to construct the pie chart but: i. did not label the value ii. the value of angles was inaccurate. iii. the calculation of angle was inaccurate.	197	125	90
Superitem 4c	Convert the stem-and-leaf plot into a dot plot	Did not understand the concept of a dot plot. i. They converted it to a line graph. ii. created their forms of data representation	Able to construct the dot plot but: i. did not label for the axis. ii. scale was incorrect iii. x-axis was incorrect	193	109	85

Table 4 (continue)

No	Content	Transformation	Process skill	Total number of students who answered partially correctly (n=256)	Total number of students who answered incorrectly (n=256)	Percentage (%)
Superitem 5c	Construct a histogram to represent the data given in the table.	Did not represent the data appropriately. They converted it to various forms of graphs and charts. For example: i. bar graph ii. line graph iii. table iv. joint graph (line + bar)	Able to construct the histogram but: i. no label for the x-axis and y-axis ii. no space from the beginning of the x-axis and y-axis were mixed up iii. placed the time on the y-axis and frequency on the x-axis	107	146	71

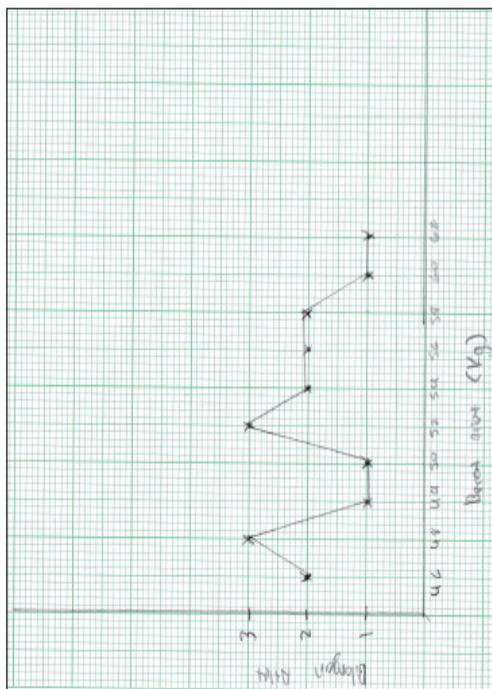


Figure 7. Example of solution 4c

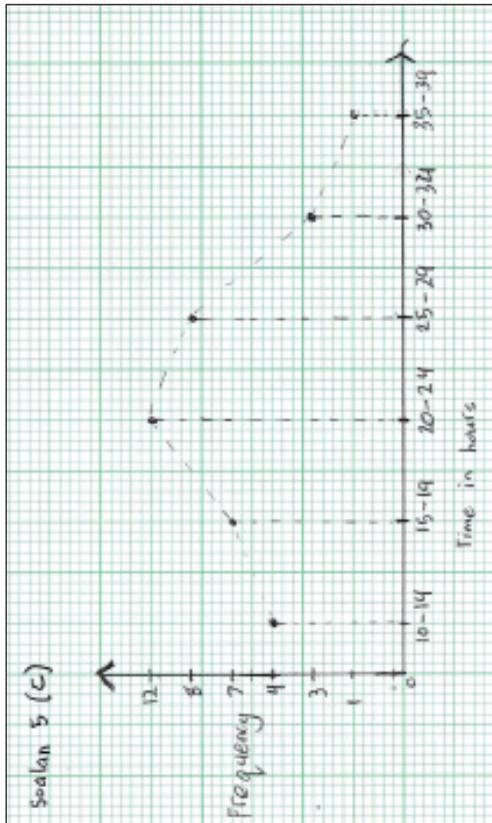


Figure 8. Example of solution 5c

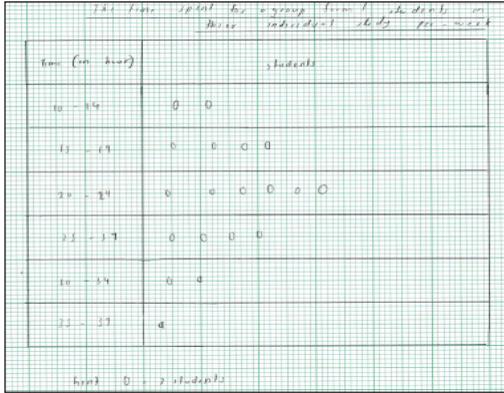


Figure 9. Example of solution 5c

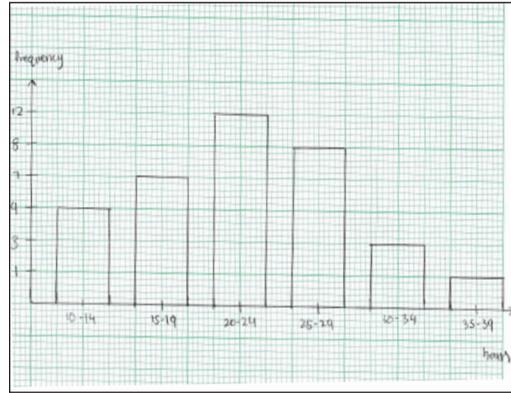


Figure 10. Example of solution 5c

failed to provide a reason or solid reason for their suggestions and opinions. The reasons provided were superficial and incomplete. They did not show their critical analysis and logical thinking of the contexts. Some students even gave responses based on their imagination without reflecting on the contexts. Figures 11 and 12 show the general reasons given by the students for Superitem 1. The students were expected to state the

constant of linear patterns identified from their line graph. Figures 13 and 14 show that the students could not apply logical thinking to express their justification based on the contexts. Figure 15 shows the students only stated the types of sports without providing their reasons. Meanwhile, Figure 16 shows the students' failure to provide logical reasons by relating them to the context.

Can you extend your line of graph to predict the enrolment for year 2020? Give a reason.
Yes, because the students every year increase.

Figure 11. Example of solution for 1d

Can you extend your line of graph to predict the enrolment for year 2020? Give a reason.
Yes, it is because the enrolment for year 2019 will be more than 2019.

Figure 12. Example of solution 1d

There is different quantity of cars sold for the four Perodua models. Give your reason.
Yes because the data show the different thousands of cars sold in 2017.

Figure 13. Example of solution 3d

Time spent in the individual study is the main factor for success in academic achievement. Do you agree? Explain your answer.
Yes, this is because studying makes you clever and study hard.

Figure 14. Example of solution 4d

What type of sport are normally involved by athletes whose weight more than 54 kg? Give your reason.
Weight lifting.

Figure 15. Example of solution 5d

Some people concluded that the school must be located in town. Do you agree with this opinion? Give a reason.
Yes I agree it is because there has a bus school.

Figure 16. Example of solution 2d

Table 5
Common error analysis for extended abstract level items

No	Content	Process skill	Encoding	Total number of students who answered partially correctly (n=256)	Total number of students who answered incorrectly (n=256)	Percentage (%)
Superitem 1d	Predict the enrolment for the year 2020. Give a reason.	Did not extend the line graph to show the prediction.	Did not show the statistical reasoning by applying the mathematics concept to support the explanation.	233	83	89
			Did not make the connection between the prediction and the line graph. The reasons were inaccurate and incomplete. For example: i. add 50 ii. it is linear iii. it is a straight line iv. increasing v. more students will enrol in the year vi. they are more people vii. the number of registered students will be the same every year. viii. because it is the same			
Superitem 2d	Give an opinion of the trend and justify it.		Give reason based on their imagination without reflecting on the contexts. The reason was superficial and incomplete. For example: i. it is good ii. more people live in town iii. a lot of rich men iv. town is modern v. has many people vi. school in a town with traffic jam vii. the village has no school viii. prefer using public transport in town ix. most of them use the bus x. because most of them use cars, buses, and motorcycles to school xi. because 83% of students use the vehicle to school xii. because 83% of students use the vehicle to school	252	95	97

Table 5 (continue)

No	Content	Process skill	Encoding	Total number of students who answered partially correctly (n=256)	Total number of students who answered incorrectly (n=256)	Percentage (%)
			<ul style="list-style-type: none"> xiii. majority going to school by bus xiv. has school bus xv. there are many facilities xvi. more people take buses and cars compared with others xvii. students use many kinds of transport 	70	267	94
Supertitem 3d	<p>There is a different quantity of cars sold for the four Perodua models. Give your reason</p>		<p>The reason did not show their logical thinking. The reason given was not linked to the context shown. Give a brief reason for the best-selling car whereas the question required the reason about the different quantity of car sold. The reason was superficial and incomplete. For example:</p> <ul style="list-style-type: none"> i. Axia is the cheapest ii. Axia is cheap iii. The best car iv. Different taste v. The price of the car is different vi. Each model has different features vii. Axia saves petrol viii. The demand for different brands was different 	167	77	69
Supertitem 4d	<p>What type of sport normally involves athletes whose weight is more than 54kg? Give your reason.</p>		<p>Did not provide the reason. The reason given was not linked to the context shown. The reason was superficial and incomplete. For example:</p> <ul style="list-style-type: none"> i. Wrestling because they are strong ii. Shot put because more stamina needed iii. Weightlifting because need more energy 	255	47	85
Supertitem 5d	<p>Time spent in individual study is the main factor for success in academic achievement. Do you agree? Explain your answer.</p>		<p>Failure to provide reasons. The reason given was not linked to the context shown. The reason was superficial and incomplete. For example:</p> <ul style="list-style-type: none"> i. More knowledge you know ii. The grade is important to us iii. We can succeed in academic iv. We need to do a revision v. Study can make you clever vi. Study can get you to discipline 			

DISCUSSION

This study analysed the students' errors in solving statistical superitem tasks. As stated in Newman's error analysis, the result showed four stages of common errors: comprehension, transformation, process skill, and encoding. The students achieved the reading stage successfully. This stage only involved basic statistical skills as the students were expected to read the data and information provided. The results of this study were consistent with previous studies conducted by Erna and Budi (2016) and Fitriani et al. (2018). They argued that the other four stages of errors are students' most common errors in solving mathematical tasks.

The students made the least errors in solving uni-structural level items except for Superitems 3 and 4. The lack of understanding of the data representation form was an error made by the students. Even though they showed their ability to read the problem, they failed to understand the data highlighted in the task. In Superitem 4, although the item is very simple, namely identifying the number of athletes who weigh 46 kilograms (kg) by referring directly to the stem-and-leaf plot shown, some students showed their inability to identify what was required by the task. They had difficulty referring to the correct information from the stem-and-leaf plot to respond correctly (Fitriani et al., 2018; Wijaya et al., 2014).

Almost all the students responded correctly to the multi-structural level items in Superitems 1 and 3. They were

required to find the difference between two values and identify the highest number, respectively. However, the students had difficulties calculating the mean values in Superitems 2 and 5. As a result, they made obvious errors in the transformation and process skills stages. The students generally made mistakes in carrying out the operations. For instance, while calculating the mean value, they failed to select the appropriate mathematical operation to get the mean value. Furthermore, they tended to use addition, subtraction, and division rather than multiplication. As a result, they made transformation errors in these items. Although some students could select the appropriate operation in calculating the mean value, the process skill errors hindered them from arriving at the correct responses. This problem is similar to previous studies, which revealed that the students had difficulties interpreting the data, especially in carrying out the appropriate operation to determine the mean value (Ozmen et al., 2020; Yun et al., 2016). This finding was also supported by Idehen (2020) and Ishaku and Idris (2017). They noticed that the main factor influencing this problem was the lack of understanding of the basic concept of statistics, namely central tendency. It was also categorised as a mechanical error whereby the students were always trained to follow the formula without understanding the underlying principle.

Most of the students showed their inability to represent the data in a histogram, dot plot, and pie chart, namely 71%, 85%, and 90%, respectively of the students failed to gain a full score for the relational level

items. In addition, they made transformation and process skill errors when they failed to represent the data with histograms, dot plots, and pie charts. Meanwhile, 41% and 46% of students failed to accurately represent the data with line and bar graphs, respectively. Ozmen et al. (2020) and Yayla and Ozsevgec (2015) claimed that the students normally have lower success in constructing graphs than in reading and interpreting the graphs. It might be because the students were not given enough time to practise in the classroom.

Although all forms of data representation were highlighted in the mathematics curriculum, the students performed more successfully constructing bar and line graphs. Watson (2006) stated that the students frequently encountered both types of graphs in their books and mass media. Thus, they were more familiar with the graphs and managed to display them correctly. Similarly, Capraro et al. (2005) and García-García and Dolores-Flores (2021) also found that most students constructed the graph they were familiar with or were their favourite. Yun et al. (2016) found that the students' successes depended on the different representation forms. They might perform better in constructing the bar and line graphs but were unsuccessful in the histogram and dot plot. This result might stem from the confusion about the various forms of data representation. The histogram and bar graphs were the most prominent confusion in this study.

On the other hand, some students could construct the graph correctly, but they made process skills errors. For instance, they did

not label the axis and its scales correctly. Yun et al. (2016) revealed that the students saw the construction of the graph as the final product of learning the topic with little idea of its interpretability. As a result, they always faced problems interpreting and analysing their graphs. Friel et al. (2001) and Idehen (2020) drew attention to these types of errors in their study. They stated that the main factor causing these errors was insufficient statistical knowledge related to naming the scale and axis. Scaling was found as the most serious problem faced by the students.

More than 80% of students could not provide complete responses for all the extended abstract level items except for Superitem 4 (69%). The encoding errors were made due to their inability to justify and make conclusions about their responses. The most influential factor is their low level of reasoning and creativity ability. The level of 'read beyond the data is the most challenging item as they were asked to make predictions, inferences, and justify the situation. Students depended on their thought to make the justification without focusing on the context of the task. Watson (2006) claimed that the students preferred to give brief and general reasons based on the informal criteria. According to Fitriani et al. (2018), this problem occurred due to the students' thinking that the most important thing they need to show their mathematical competencies is correctly getting the answer in value. They were unfamiliar with writing and expressing their justifications and logical ideas based on the data.

Meanwhile, Ozmen et al. (2020) stated that the statistical learning environment less encourages the students to develop this high level of statistical skills. Similarly, Bragdon et al. (2019) also highlighted these issues. They claimed that this failure resulted from insufficient activities encouraging students to think critically and creatively about statistics in real-life contexts.

CONCLUSION

Based on the findings of this study, it is obvious that the students made statistical errors, including misunderstandings, misconceptions, and carelessness. This problem could lead to more complex difficulties when they learn advanced levels of mathematics. Therefore, teachers must address and highlight the errors during the teaching and learning of statistics to prevent them from becoming more critical and complicated.

Newman's error analysis has helped the teachers and students with detailed information about the common statistical errors. As a result, teachers can develop a more effective teaching approach focusing on a more profound understanding of the statistics concepts. Without the proper understanding of the concepts, it is difficult for the students to generalise, predict and make inferences to solve the statistics problem (Sari & Bernard, 2020). Moreover, this study provides a meaningful analysis that assists the teaching and learning of statistics to create a better link between numeracy and literacy.

The application of multi-level tasks provides valuable information that enables

the teachers to understand better their students' statistical processes in terms of understanding the data provided, calculating and comparing the value of data, representing the data in various forms, and making inferences and predictions. More importantly, it also allows the teachers to easily detect the students' common errors at various complexity of the items in a hierarchical manner (Nasser & Lian, 2021). This information is very useful for the teachers in providing informative and specific feedback to improve students' learning process. In addition, this information will also provide opportunities for the students to reflect on their progress and identify errors and weaknesses that need to be improved. Newman's analysis provides various errors that can lead them to achieve the highest statistical and cognitive processes required.

The SOLO model has the advantage of having a hierarchical cognitive development. Therefore, it is appropriate to be applied in developing a variety of valid and reliable diagnostics assessment frameworks and instruments, not only for mathematics but also for other areas of education. Besides, the result of this study demonstrates that this model can systematically distinguish errors based on the surface or deep level of understanding. Therefore, it is especially beneficial for teachers and students to acquire early information on what needs to be addressed and improved to grasp a particular topic. Furthermore, since this study demonstrated the effectiveness of using the SOLO model to predict common error patterns, the use of this model in error

analysis can be replicated in other fields of study.

Although the findings of this study could not be applied to generalise the students' common errors in statistics, it serves as an important reference in planning and setting the teaching and learning strategies that would minimise the students' errors in statistics. For the instruction of these topics to reach their full potential, there is a pressing need to develop teaching and learning strategies that focus on the four statistical processes highlighted in the framework and build the connection between all these content domains.

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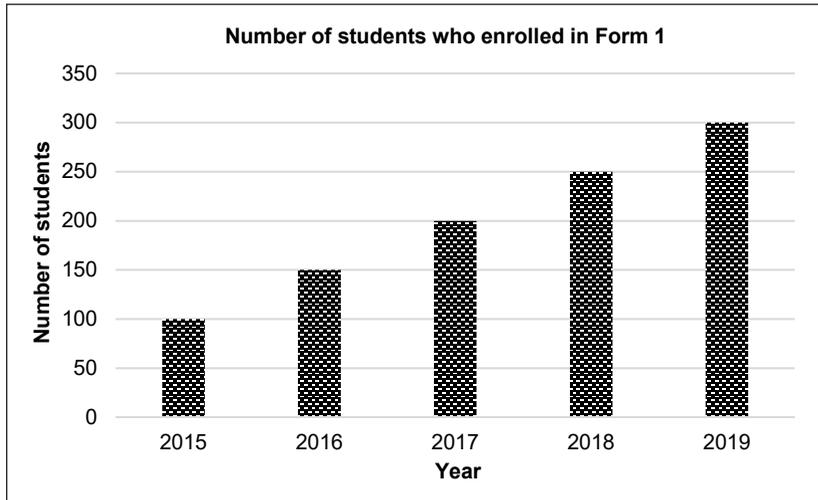
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APPENDIX

Appendix 1

The following bar chart shows the number of students who enrolled in Form One in SMK Sungai Pasir within five years.



- How many students enrolled in the year 2016?
- Find the difference in the number of students enrolled in 2015 and 2019.
- Convert the bar chart into a line graph.
- Can you extend your line graph to predict the enrolment for the year 2020? Give a reason.